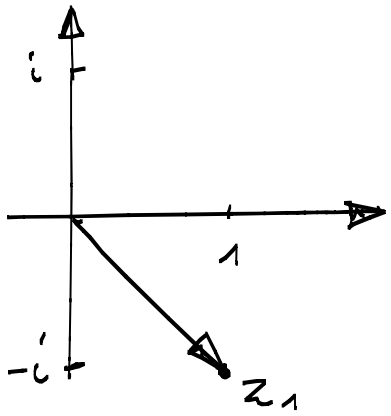
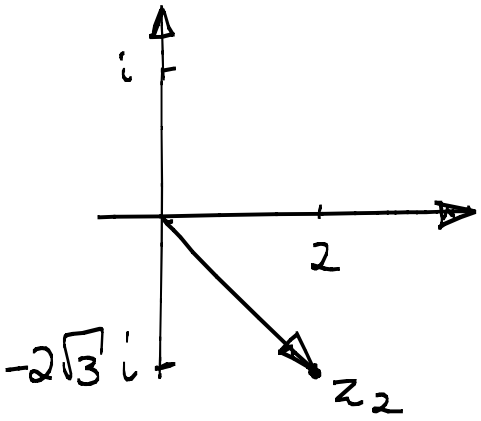
Übungen zur Mathematik 1

Lösungen Blatt 7

Aufgabe 1:

a)

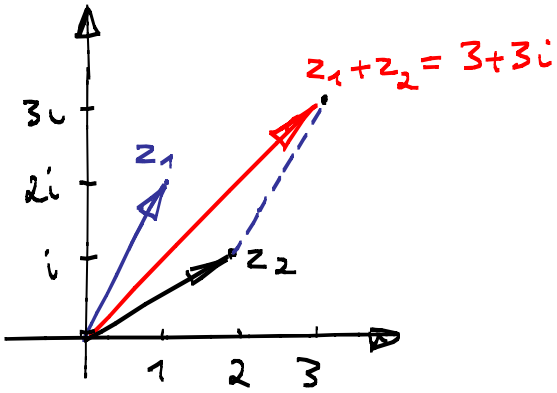


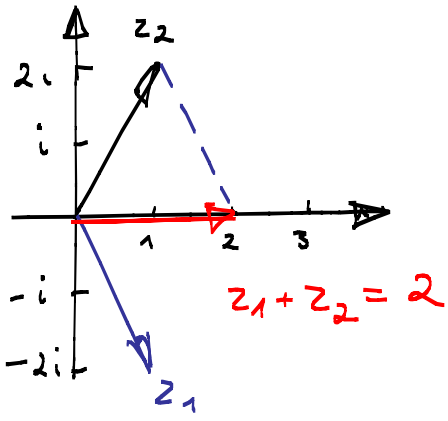


$|z_1| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$|z_2| = \sqrt{2^2 + 2^2 \cdot 3} = 4$

b)





c) $z_1 + z = 10$

$z_1 - z_2 = 6i$

$z_1 \cdot z_2 = 5^2 - (3i)^2 = 25 - 9(-1) = 34$

$$\frac{z_1}{z_2} = \frac{5+3i}{5-3i} = \frac{(5+3i)(5+3i)}{(5-3i)(5+3i)} = \frac{25+30i+9i^2}{34}$$

$$= \frac{16+30}{34} = \frac{8}{17} + \frac{15}{17}i$$

$|z_1| = \sqrt{25+9} = \sqrt{34}$

Aufgabe 2

a) $15+20i+6i-8 = 7+26i$

b) $8-10i+12i+15 = 23+2i$

c) $1+2\sqrt{2}i+2i^2 = -1+2\sqrt{2}i$

d) $9-6\sqrt{5}i+5i^2 = 4-6\sqrt{5}i$

e) $\frac{(6-2i)(6-2i)}{(6+2i)(6-2i)} = \frac{36-24i+4i^2}{36-4i^2} = \frac{32-24i}{40} = \frac{4}{5}-\frac{3}{5}i$

f) $\frac{(56+33i)(12+5i)}{(12-5i)(12+5i)} = \frac{672+280i+396i-165}{144+25}$

$= \frac{507+676i}{169} = 3+4i$

g) $\frac{(\sqrt{3}-\sqrt{2}i)(\sqrt{3}-\sqrt{2}i)}{(\sqrt{3}+\sqrt{2}i)(\sqrt{3}-\sqrt{2}i)} = \frac{3-2\sqrt{2}\sqrt{3}i-2}{3+2} = \frac{1}{5}-\frac{2\sqrt{6}}{5}i$

h) $\frac{5i(\sqrt{2}+\sqrt{3}i)}{2+3} = \frac{-5\sqrt{3}+5\cdot\sqrt{2}i}{5} = -\sqrt{3}+\sqrt{2}i$

Aufgabe 3

a) $(2+3i)(4-2i) = 8-4i+12i+6 = 14+8i$

$\overline{(2+3i)}\,\overline{(4-2i)} = (2-3i)(4+2i) = 14-8i$

Es gilt also $\overline{z}_1 \cdot \overline{z}_2 = \overline{z_1 \cdot z_2}$

b) $\frac{2+3i}{4-2i} = \frac{(2+3i)(4+2i)}{16+4} = \frac{2+16i}{20} = \frac{1}{10}+\frac{4}{5}i$

$\frac{2-3i}{4+2i} = \frac{(2-3i)(4-2i)}{16+4} = \frac{2-16i}{20} = \frac{1}{10}-\frac{4}{5}i$

Es gilt also $\overline{z_1} : \overline{z_2} = \overline{z_1 : z_2}$

c) $|z_1| = \sqrt{4+9} = \sqrt{13}$
$|\overline{z_1}| = \sqrt{4+9} = \sqrt{13}$
} z_1 und $\overline{z_1}$ haben denselben Betrag

Aufgabe 4

a) $z_1 + z_2 z_3 = 5 - 3i + (2+4i)(-3-6i)$

$= 5 - 3i + 18 - 24i$

$= 23 - 27i$

b) $z_1^2 z_2 = (16 - 30i)(2+4i) = 152 + 4i$

c) $\frac{z_1 z_2}{z_3} = \frac{(5-3i)(2+4i)}{-3-6i} = \frac{22+14i}{-3-6i}$

$= \frac{(22+14i)(-3+6i)}{9+36} = \frac{-150+90i}{45}$

$= -\frac{10}{3} + 2i$

d) $z_1 + \frac{z_2 z_3}{z_1 + z_3} = 5 - 3i + \frac{18-24i}{-1-2i}$

$= 5 - 3i + \frac{(18-24i)(-1+2i)}{5}$

$= 5 - 3i + \frac{30+60i}{5}$

$= 11 + 9i$

e) $\dfrac{2+4i}{(5-3i)(-3-6i)} = \dfrac{2+4i}{-33-21i}$

$$= \frac{(2+4i)(-33+21i)}{33^2+21^2}$$

$$= \frac{-150-90i}{1530}$$

$$= -\frac{15}{153} - \frac{9}{153}i$$

$$= -\frac{5}{51} - \frac{1}{17}i$$